

Simulation of viscoelastic flows at high Weissenberg number using a generic numerical stabilization framework

Matthias Niethammer¹, Holger Marschall¹, Christian Kunkelmann², Dieter Bothe¹

¹ Mathematical Modeling and Analysis, Technische Universität Darmstadt, Germany ² GCP/RC - BASF SE, Ludwigshafen, Germany

niethammer@tfi.tu-darmstadt.de, marschall@tfi.tu-darmstadt.de, christian.kunkelmann@basf.com, bothe@tfi.tu-darmstadt.de

The numerical simulation of viscoelastic flows in complex geometries is challenging due to the presence of geometric singularities and sharp stress boundary layers which occur in the high Weissenberg number limit [1]. The 'High Weissenberg Number Problem' (HWNP) has been a major issue in computational rheology for more than four decades [2]. It refers to the incapability of numerical methods to resolve these geometric singularities and boundary layers, causing all computations to break down at relatively low critical values of the fluid elasticity. Although a complete solution is not known until today, several effective stabilization methods have been developed to cope with the HWNP [3, 4, 5]. However, there is a multitude of constitutive models describing viscoelastic material behavior, with which the stabilization approaches are to be combined. This combination has been realized in a general way by deriving model-independent forms of the stabilized equations.

We have developed a robust finite volume method for viscoelastic flow analysis on general unstructured meshes. It is build upon a new general-purpose stabilization framework for high Weissenberg number flows. The numerical framework provides full combinatorial flexibility between different kinds of rheological models on the one hand, and effective stabilization methods on the other hand. A special face interpolation technique is employed to the cell-face interpolation of the stress in the divergence operator of the momentum balance to remove the decoupling between the velocity and stress fields, arising from the collocated variable arrangement. The discretized system of equations is solved in a segregated solution approach. The numerical methods have been implemented by massive use of generic C++ template programming, runtime polymorphism and overloading.

By means of evaluations of established benchmark-tests, such as the entry-flow of Oldroyd-B and PTT fluids through a 4:1 contraction, we demonstrate that the numerical methods are robust over a wide range of Weissenberg numbers and significantly alleviate the HWNP [6]. The accuracy of the results is evaluated in a detailed mesh convergence study. Having a set of different stabilization approaches available on the same computational platform, we quantitatively investigate their impact on mesh-convergence and mesh-sensitivity, thus benchmarking the numerical code and yielding new understanding of viscoelastic entry-flows at higher Weissenberg numbers.



References

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