Adjoint optimization with thermal constraints

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The problem of the optimization of aerodynamic shapes is the research of the best configuration in order to obtain a certain objective, respecting external constraints. An optimization problem is generally stated as the problem of the minimization of an objective function. Besides the most classical methods that test randomly (or almost) the different configurations, nowadays the use of techniques that combine CFD and numerical optimization routines is increasing [1]. Among the different techniques, the adjoint strategy represents an efficient approach to the optimization problems when a large number of design variables is involved. It is based on the computation of a sensitivity map in the domain, representing how to variate the design variables to get the optimal configuration and leading to automatic modifications of geometry.

The adjoint optimization, already widely used for the reduction of the pressure losses, is considered in this work in order to achieve a geometry enhancing mixing by prescribing a uniform temperature distribution at the outlet section as the objective function. The application of this method to problems with thermal constraints is not widely studied, but its potentialities are evident. In this work, a 2D and a 3D test case are studied in the OpenFOAM framework.

Introduction

The adjoint optimization method is based on the definition of the adjoint flow variables, that are directly related to the sensitivity, giving an indication on the direction of improvement. The new variables are computed solving a system of equations, having a structure similar to Navier-Stokes equations. Hence, the cost for solving the problem is equivalent to solving two N-S systems, independently from the number of design variables chosen. The continuous formulation of the method is considered in this work [2]: OpenFOAM offers object-oriented implementation of differential operators, that makes the implementation of a continuous adjoint very straightforward [3]. Moreover, it is possible to formulate the continuous adjoint equations in a way that can be quickly adapted to the different objective functions.

The procedure applied is the topological shape modification: the fluid domain is considered as porous, where the porosity ϵ is defined as the fluid volume fraction. The variable is introduced in the momentum equation, through a Darcy term, by means of a value of permeability, that is maximum where the porosity is 1. In the thermal equation, the dependence from the porosity is in the definition of the thermal diffusivity. In the topological optimization process, the porosity field is considered as design variable. Modifications of the design towards the optimal configuration are obtained updating the porosity field through a steepest descent method that uses the value of sensitivity δ_L of each cell. In this way, a new configuration of the topology is defined. The regions with low value of porosity block completely or partially block the passage of the flow, according to the maximum value of the inverse permeability imposed as constraint in the case setup.

In OpenFOAM a solver for the topological optimization is already present, called *adjointShapeOptimizationFoam*. It includes solving procedures for primal and adjoint equations system for the momentum and mass conservation, as well as a topological shape modification algorithm. It represents the base solver to obtain optimized geometry that minimize objective functions dependent on the pressure and the velocity fields. Examples of applications are the pressure loss minimization or the retrieval of a uniform velocity profile at the outlet. The current solver does not take into account temperature variations.

In order to extend the solver capability and to gain the possibility of defining objective functions that involve the temperature, the existing solver is modified. The thermal equation is introduced both for the primal and for the adjoint system. Moreover, the existing equations and the relative BCs are modified in order to introduce the dependence from the temperature field and the new objective function [4]. The equations implemented to compute the adjoint variables are:

$$\nabla \cdot u + \frac{\partial J_\Omega}{\partial p} = 0$$

$$\nabla u \cdot v - (v \cdot \nabla) u + \alpha(\epsilon) u - \nabla(2\nu D(u)) + \nabla q - \varphi \nabla T + \frac{\partial J_\Omega}{\partial v} = 0$$

$$-v \cdot \nabla \varphi - \nabla \cdot (K_t(\epsilon) \nabla \varphi) + \frac{\partial J_\Omega}{\partial T} = 0$$

Where: (v, p, T) are the velocity, pressure and temperature of the primal flow; (u, q, φ) are the adjoint velocity, adjoint pressure and adjoint temperature; ν is the kinematic viscosity; $D(u) = \frac{1}{2}(\nabla u + (\nabla u)^T)$; J represents the objective function; K_t and α represent the thermal diffusivity and the inverse permeability respectively. Since the fluid and the solid zones have different values of thermal diffusivity, K_t is computed as a function of the porosity value and of the characteristics of the considered materials. The algorithm implemented is shown in Figure 1.

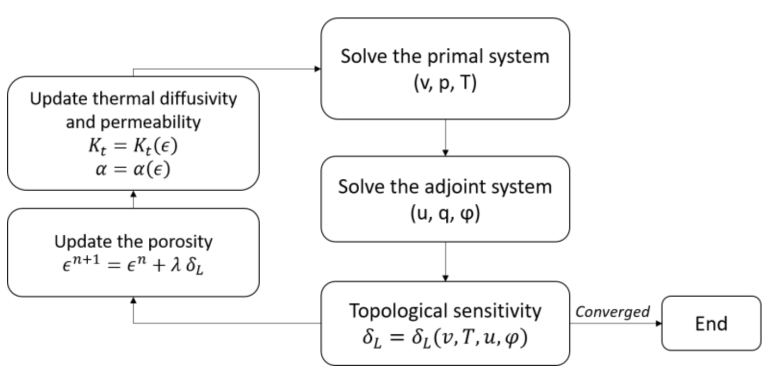

Figure 1. Adjoint solver algorithm.

The choice of the cost function with no volumetric contributions, as in the case of most of the duct flows problems, simplifies the problem, getting rid of the dependency of the equations from the cost function. This makes the equations independent of the cost function. It means that the implementation of the system does not require modifications if the cost function is changed. Instead, the boundary conditions (BCs) are related to the surface contribution of the objective function and they represent the only dependence of the problem from the cost function imposed. Hence, the BCs need to be modified when the objective function is changed.

The objective function introduced is the uniformity of the temperature at the outlet section:

$$J = \int_{\Gamma} (T - T_d)^2 d\Gamma$$

where T_d is the "design temperature", i.e. the objective temperature to reach on the interested section. It is user-defined at the beginning of the numerical process.

While in the case of the pressure loss minimization the porous area describes the new shape of the geometry optimized, in the case of the thermal applications, the porous material is influencing both the velocity distribution and the heat transfer process.

Test case optimization in 2D domain

The test case taken into account is represented in Figure 2. The Reynolds number considered is Re = 350, obtaining a laminar regime. The upper and the lower walls are respectively cooled and heated for ¾ of the length at a temperature of 243 K and 343 K, while the inlet temperature is 293 K.

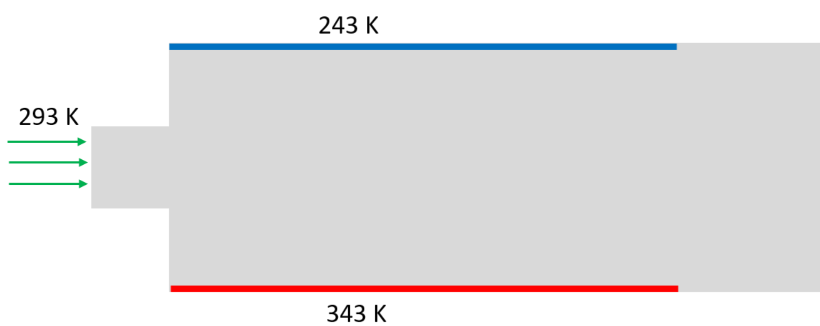

Figure 2. Schematic representation of the temperature boundary conditions.

Solving the case with a SIMPLE algorithm that includes also the thermal equation, the temperature distribution obtained is shown in Figure 3.

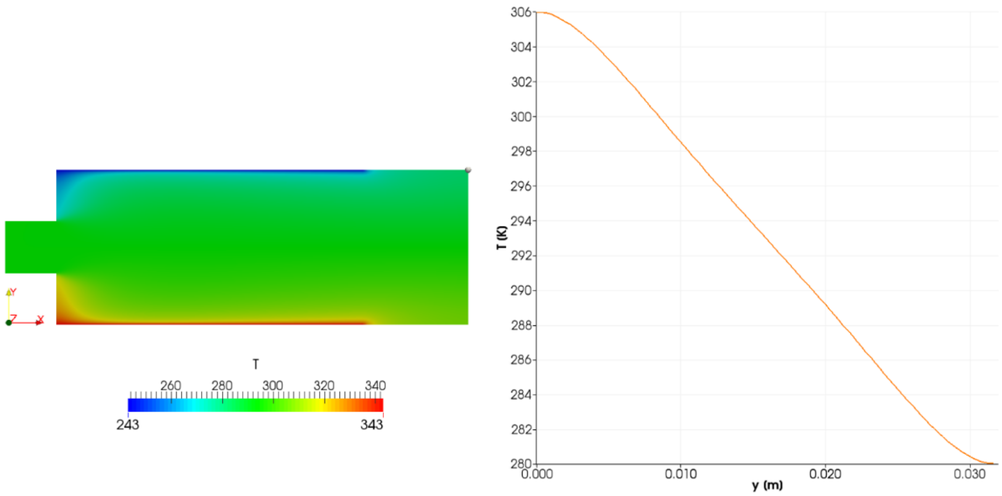

Figure 3. Temperature distribution of the test case and temperature profile along the outlet section.

Starting from the results of this simulations, the optimization process is performed with a design temperature at the outlet of $T_d = 320\ K$. Moreover, it is necessary to impose the values of the thermal diffusivity both for the fluid part, air in the present case, and the solid part, hence where the porosity is fixed to 0 and no air is present. The values for solid and fluid diffusivity are respectively $K_s = 8.33e - 04\frac{m^2}{s}$ and $K_f = 1.68e - 05\ \frac{m^2}{s}$. The maximum value of inverse permeability is set to $\alpha_{Max} = 200\ s^{-1}$ and corresponds to the zone identified as solid.

The outcome of the optimization process is shown in Figure 4. The red area, characterized by the maximum value of inverse permeability α, represents the solid zone. After the optimization, the outlet temperature reaches a maximum value of around 315 K with a distribution that is close to uniform for the most of the section.

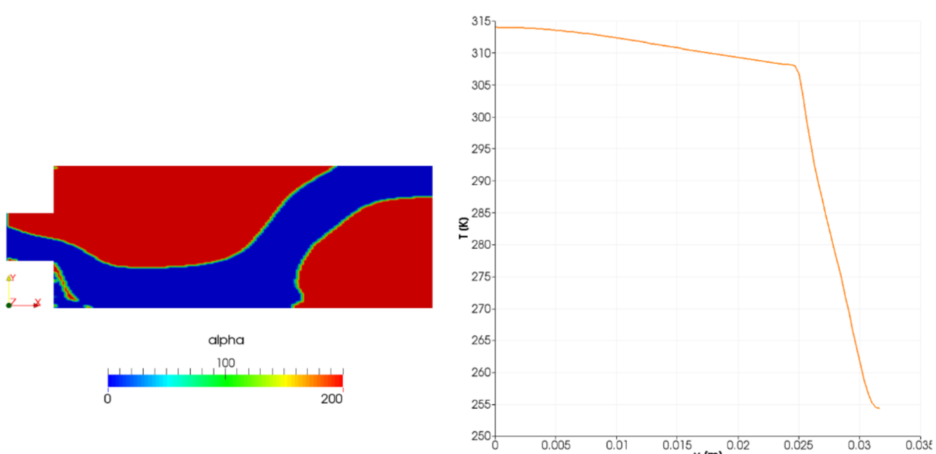

Figure 4. Inverse permeability distribution after the optimization process and T profile along the outlet section.

Because the inlet temperature is lower than T_d, a porosity region is created in proximity of the cooled wall, forcing the passage towards the heated wall. This leads to a higher temperature at the outlet section than the inlet. Moreover, the presence of the porous material in bottom right part of the domain helps the diffusion of the high temperature to the outlet, due to the thermal diffusivity value. The distribution of the porosity is therefore in agreement with the expected results.

Test case optimization in 3D domain

The optimization of the same geometry is performed also in 3D. The same boundary conditions and setup of the 2D case are used for the 3D extension. In this case, the front and the back patch are consider as adiabatic walls.

The results obtained are shown in Figure 5. The red area represents the solid area, expressed in terms of solid fraction. The value 1 corresponds to the higher value of inverse permeability. As in the 2D case, with the new topology obtained in the 3D extension give an outlet temperature that tend to a uniform value of 315 K.

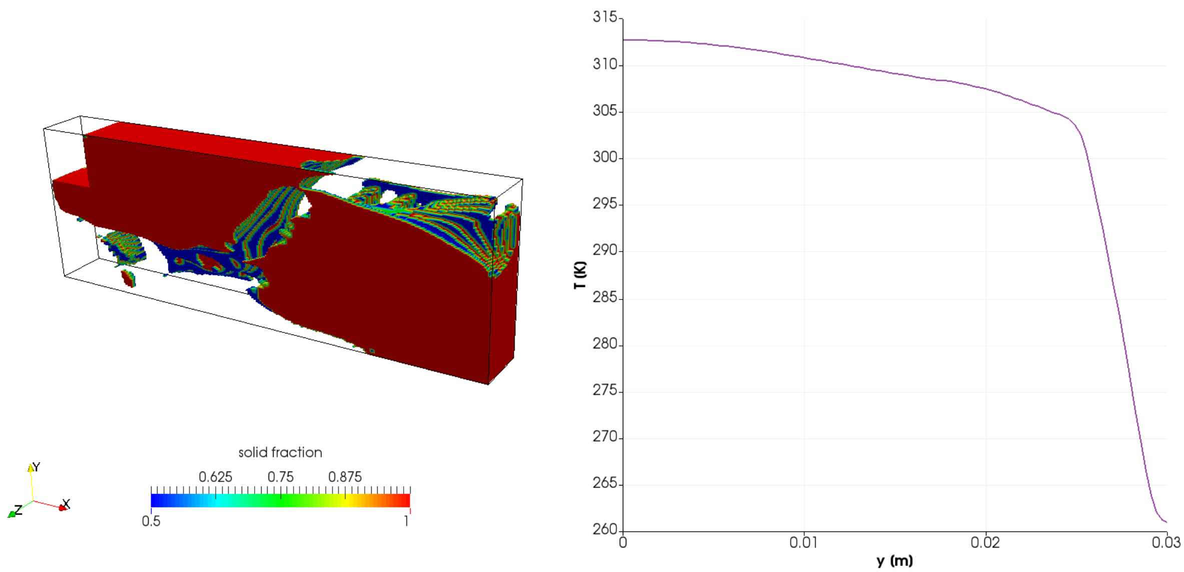

Figure 5. Solid fraction distribution in the 3D domain and T profile along the centerline of the outlet section.

Looking at the results obtained with this technique, it is possible to conclude that the adjoint method application is very promising in case of optimization with thermal constraints, and can be extended to more complex geometries.

References

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