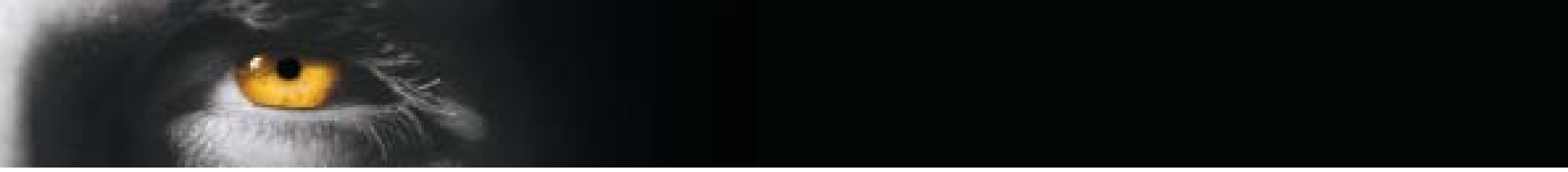


Understanding Heat Transfer

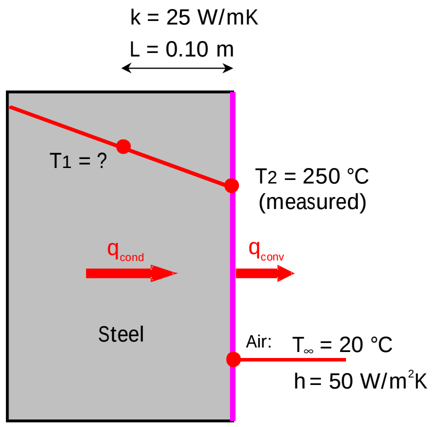
e-Tips
Nr. 4

In this e-tip edition, we will treat situations for which heat is transferred by diffusion under one-dimensional, steady-state conditions. Despite their inherent simplicity, one-dimensional, steady-state models may be used to accurately represent numerous casting situations.

The term "one-dimensional" refers to the fact that only one coordinate is needed to describe the spatial variation of the temperature. Hence, in a one-dimensional system, temperature gradients exist only in one direction and heat transfer occurs exclusively in that direction. The system is characterised by steady-state conditions if the temperature at each location is independent of time.

Example 1

Problem



A temperature of 250°C is measured on the outside surface of a steel slab. This surface is in contact with air at 20°C.

Knowing the thermal conductivity of the material (k=25 W/mK), the surface heat transfer coefficient (h=50 W/m²K), what is the steady state temperature of the metal 10 cm below the surface?

Solution

We find the solution by considering the heat equilibrium on the outside surface. Knowing that the temperature of that surface does not change with time (steady-state), we assume thermal equilibrium on it.

The heat flux "arriving" on that surface from inside is given by Fourier law:

$$k\left(\frac{T_1 - T_2}{L}\right)$$

The heat flux "leaving" the surface is:

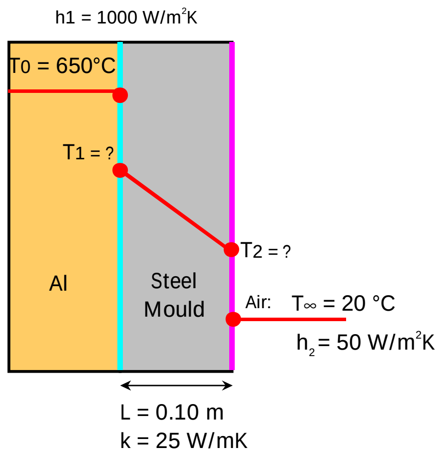
$$h\left(T_2 - T_\infty\right)$$

By equalling these two entities, it is found that: $T_1 = \frac{hL}{k}\left(T_2 - T_\infty\right) + T_2$

The answer is T_1 = 296°C.

Example 2

Problem



Aluminium, at a constant and uniform temperature of 650°C, is in contact with a steel mould of 0.1 m of thickness and with a thermal conductivity of k=25 W/mK.

The outside steel mould is in contact with air at 20°C and the interface heat transfer coefficient has a value of 1000 W/m²K.

What is the steady state temperature at both sides of the steel mould?

Solution

We find the solution by considering the heat equilibrium on the inner and outside surfaces of the steel wall. Knowing that the temperature does not change with time (steady-state), we assume thermal equilibrium on both surfaces:

Inner surface:

$$k\left(\frac{T_1 - T_2}{L}\right) = h_1\left(T_0 - T_1\right)$$

Outer surface:

$$k\left(\frac{T_1 - T_2}{L}\right) = h_2\left(T_2 - T_\infty\right)$$

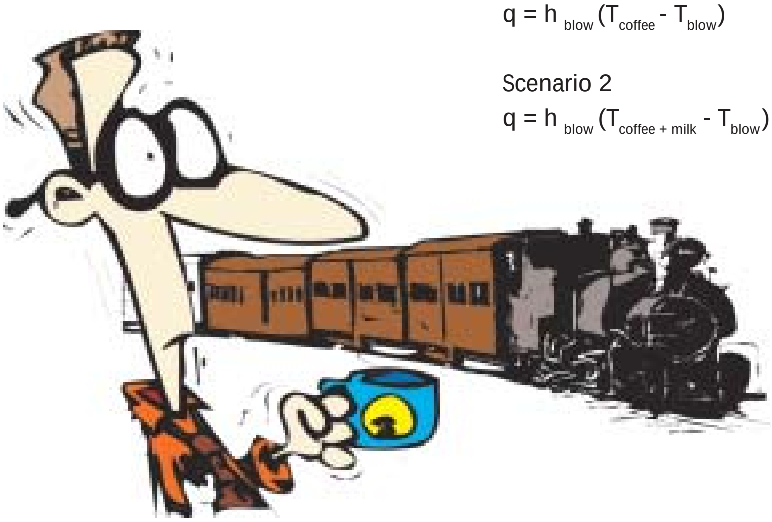
Having two equations and two unknowns we can easily find the solution.

T1 = 624°C and T2 = 524°C

If the outside convection coefficient is changed from h_2= 50 W/m²K (air cooling) to h_2= 1000 W/m²K (water cooling) we have the following result:

T1 = 528°C and T2 = 44°C

It is seen that there is a dramatic drop in the outside surface temperature (from 524°C to 44°C) but the inside surface temperature drops only by 100°C. This shows how localised the effect of cooling conditions can be.



Example 3

Problem

Now consider the following situation. A man is in a hurry to finish his cup of coffee in order to catch the departing train. To cool the coffee down to a drinkable temperature he has the possibility to add a limited amount of milk and also to blow on the surface. The milk alone is not sufficient to cool the coffee down to the required temperature nor is the blowing on the surface within the limited time. The question now arises, in order to cool the coffee down to a drinkable temperature the quickest, should he add the milk first and then start to blow (scenario 1), or should he blow first and then add the milk (scenario 2)?

Solution

The heat flux generated by blowing on the surface of the cup of coffee is defined by the following equations:

Scenario 1
$q = h_{blow} (T_{coffee} - T_{blow})$

Scenario 2
$q = h_{blow} (T_{coffee + milk} - T_{blow})$

It is clearly evident that $T_{coffee} > T_{coffee + milk}$ and then the heat flux is also greater.

Therefore, blow first and add the milk later (scenario 1).

If we assume that :

- there is no heat loss through the cup and that the temperature of the coffee is homogenous;
- the quantity of coffee is 2 dl at 85°C and milk is 0.4 dl at 20°C;
- the air temperature is 20°C and the convection coefficient of blowing is h_{blow}= 500 W/m²K;
- the specific heat capacities of the milk and coffee are the same (C_p = 3.8 kJ/kgK);
- the available time for catching the departing train is 240s;
- the exposed surface area of the cup is, A = 1.96e-03 m² (d_{cup} = 50 mm).

Then, by integrating the convection equation, $dQ = h_{blow} A (T_{coffee} - T_{blow}) dT$ and the energy equation, $dQ = m C_p dT$, we can calculate the final temperature of the coffee and milk for both scenarios 1 & 2.

Scenario 1 : $T_{final\ (coffee + milk)}$ = 59.7 °C

Scenario 2 : $T_{final\ (coffee + milk)}$ = 61.8 °C

calcom

Calcom SA
Parc Scientifique EPFL, CH-1015 Lausanne, Switzerland
phone: +41 21 693 2918 / fax: +41 21 693 4740
mail@calcom.ch / www.calcom.ch