



# Micro-segregation Length Scales

During solidification, the solid/liquid interface rejects solute into the liquid (we consider here the frequent case when the solubility of the solute element in the solid is smaller than in the liquid). This will always lead to concentration variations in the solidified alloy, known as micro-segregation. The understanding of this phenomenon is the key to interpreting the influence of solidification on the mechanical properties of cast products. Modeling is greatly facilitated by distinguishing between important factors and non-important ones.

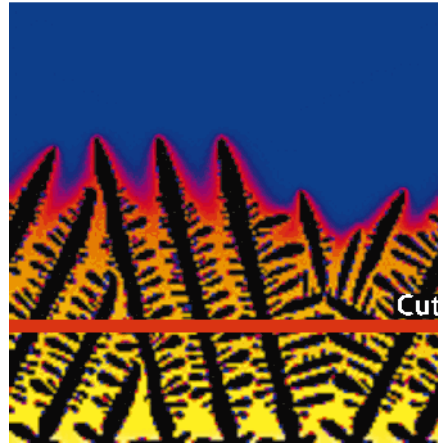


Figure a): Longitudinal representation of growing dendrites

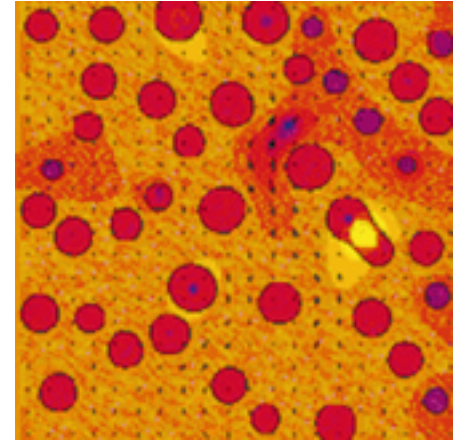


Figure b) Transversal representation of growing dendrites (red line in Figure a).

From Prof. C. Beckermann, Solidification Course 2003, EPFL-Calcom

## ◆ Case

An Al-2%Cu alloy is solidifying unidirectionally at a constant velocity  $V_L$ . The temperature gradient  $G$  is imposed by the experimental configuration and we consider for simplicity that there is no flow. The primary dendrite arm spacing is defined as  $\lambda$ .

We will use the values as defined in Table 1 for illustration:

Number	Symbol	Value	Unit	Name
1	$V_L$	0.1	mm/s	Isotherm speed
2	$G$	1000	K/m	Thermal gradient
3	$k$	0.14	-	Equilibrium distribution coefficient
4	$m$	-2.6	K/wt%	Liquidus slope
5	$T_o$	660	°C	Melting point of pure Al
6	$T_{EUT}$	574	°C	Eutectic temperature
7	$C_o$	2	wt%	Alloy concentration
8	$C_{EUT}$	33.1	wt%	Eutectic concentration
9	$\lambda$	500	$\mu\text{m}$	Primary dendrite arm spacing
10	$D_L$	$3 \cdot 10^{-9}$	$\text{m}^2/\text{s}$	Diffusion coefficient in liquid
11	$D_S$	$3 \cdot 10^{-12}$	$\text{m}^2/\text{s}$	Diffusion coefficient in solid
12	$\alpha$	$3.7 \cdot 10^{-5}$	$\text{m}^2/\text{s}$	Thermal diffusivity

Table 1:

Values 1-2, 9 are given by the experiment, they should be measured.

Values 3-6, 8 are given by the phase diagram of the Al-Cu system.

Value 7 is an input of the problem.

Values 10-12 should be found in reference books.



## Results

### 1) Liquidus temperature, $T_L$

The liquidus temperature can be calculated as follow:

$$T_L = T_o + m \cdot C_o = 654.8 \text{ }^\circ\text{C}$$

### 2) Mushy zone length, $L_M$

The mushy zone length can be calculated as follow:

$$L_M = (T_L - T_{EUT}) / G = 0.0808 \text{ [m]} \\ = 8.08 \text{ [cm]}, \text{ not so small!}$$

### 3) Local solidification time, $t_s$

The local solidification time can be calculated as follow:

$$t_s = L_M / V_L = 808 \text{ [s]} = 13.5 \text{ minutes}$$

### 4) Mean transversal speed of the solid/liquid interface in Figure b), $V_T$

The mean speed of the s/l interface in view b) can be calculated as follow:

$$V_T = (\lambda/2) / t_s = 3.1 \cdot 10^{-7} \text{ [m/s]} \\ = 0.31 \text{ [\mu m/s]} \ll V_L$$

*The transversal growth speed is much smaller than the longitudinal growth rate.*

### 5) Estimation of the thermal and solutal diffusion lengths in the liquid (Figure b)

$$L_{THERMAL} = \alpha / V_T = 119.7 \text{ [m]} \gg \lambda \\ L_{SOLUTAL} = D_L / V_T = 9.7 \text{ [mm]} \gg \lambda$$

*This means that the temperature in the liquid can be considered as uniform as well as the concentration (complete mixing). Note that the thermal diffusion length is 10'000 larger than the solutal one.*

### 6) Estimation of the thermal and solutal diffusion lengths ahead of the primary dendrite tips

$$L_{THERMAL} = \alpha / V_L = 0.37 \text{ [m]} \gg \lambda \\ L_{SOLUTAL} = D_L / V_L = 2 \cdot 10^{-5} \text{ [m]} \ll \lambda$$

*The solutal gradient ahead of the primary dendrite tips cannot be neglected.*

### 7) Estimation of the characteristic solutal diffusion times in the liquid and solid on the scale of dendrite arm spacing (Figure b))

$$t_{LIQUID} = (\lambda/2)^2 / D_L = 20.8 \text{ [s]} \ll t_s \\ t_{SOLID} = (\lambda/2)^2 / D_S = 2 \cdot 10^5 \text{ [s]} \gg t_s$$

*This means that there is enough time to smooth concentrations profiles in the liquid while this is not the case in the solid, i.e. a gradient in the solid will be present after solidification.*

### 8) Estimation of the characteristic solutal diffusion time in the liquid over the length of the mush

$$t_{LIQUID} = (L_M)^2 / D_L = 2.2 \cdot 10^6 \text{ [s]} \gg t_s$$

*This means that solutal diffusion in the liquid parallel to dendrites can be neglected because the characteristic time is much greater than the solidification time*

## Conclusion

Knowing the importance of these "orders of magnitude" influences on solidification can help us to distinguish between important factors and less important factors. These can then help us develop more simplified, but realistic, computer models of solidification phenomena while saving a lot of calculation time.