Why it is useful to describe problems in terms of non-dimensional parameters and which ones are the main important in solidification ?

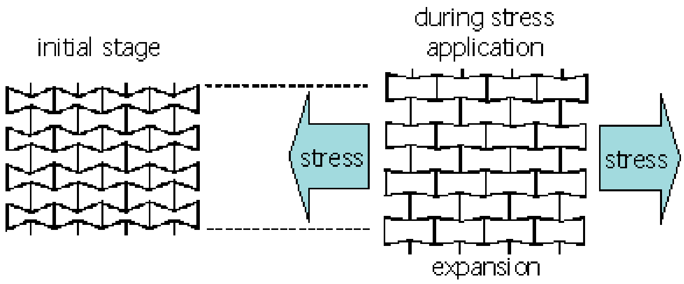
Introduction

A dimensionless quantity is a quantity without any physical units. Such a number is typically defined as a product or ratio of quantities which do have units, in such a way that all units cancel. Dimensionless quantities are widely used in the fields of physics and engineering but also in every day life.

Every one has already heard of the Mach number, a dimensionless number defined as the ratio of the speed of an object (relative to a fluid medium) divided by the speed of sound (in that medium). As the Mach number is defined as the ratio of two speeds, it is a dimensionless number. So, an aircraft travelling at Mach 1 is travelling at a speed of 340 m/s, the speed of sound in air.

Similarly, every one has already heard of the drag coefficient Cx, which is a common metric in automotive design. The drag coefficient is a dimensionless quantity that describes a characteristic amount of drag caused by air and used in the drag equation. Two cars of the same frontal area moving at the same speed will experience a drag force proportional to their Cx numbers. Automotive designers are therefore striving to achieve low drag coefficients to improve fuel efficiency at high speeds. Reducing drag is also a factor in sports car design where fuel efficiency is less of a factor but where low drags help a car to achieve a high top speed.

Finally all engineers know the Poisson's ratio, another dimensionless number. When a sample of material is stretched in one direction, it tends to get thinner in the other two directions. Poisson's ratio is a measure of this tendency: it is the ratio of the relative contraction strain divided by the relative extension strain. Most practical engineering materials have a Poisson's ratio between 0.0 and 0.5. Cork for wine bottles (mainly used by engineers during meals, especially in France) is close to 0.0, most metallic materials are around 0.3 and rubber is almost 0.5. Some polymer foams have a negative Poisson's ratio, i.e. if these materials are stretched in one direction they become thicker in the perpendicular directions.



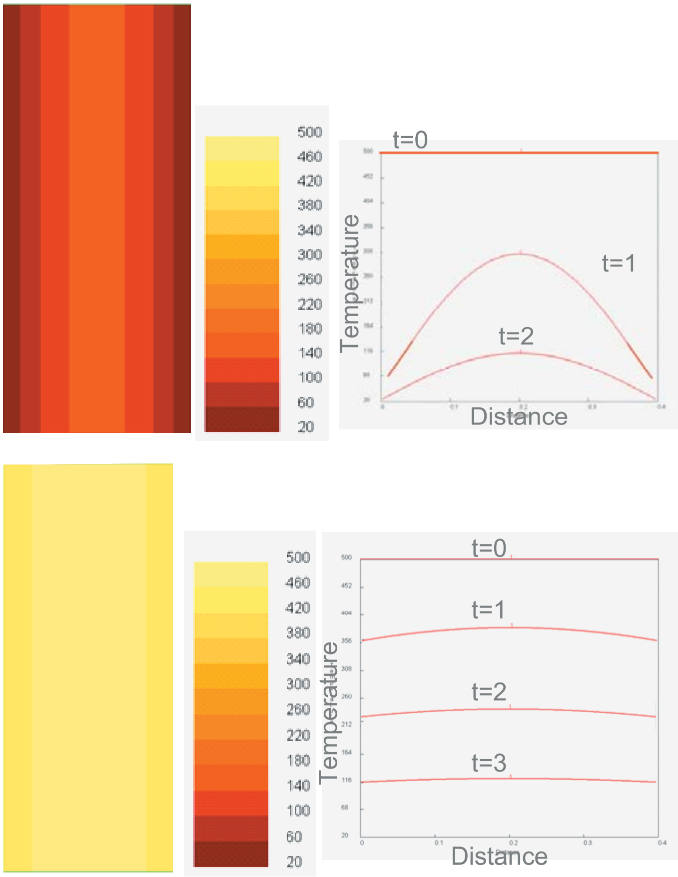
Example of a material with a negative Poisson's ratio. Stretching of normal honeycomb, shown here, illustrates the concept.

Some Speficic Dimensionless Quantities

There are many dimensionless quantities but we will focus in this etip only on two numbers very useful when addressing solidification problems: the Biot number and the Fourier number.

The Biot Number

The Biot number (Bi) is a dimensionless number used in unsteady-state heat transfer calculations. It provides a measure of the temperature drop in a solid relative to the temperature difference between the surface and an external medium like air. If the Biot number is much smaller than 1, it is reasonable to assume a uniform temperature distribution across the solid at any time during cooling (no thermal gradient, picture 1, lower part). The Biot number is defined as the ratio of hL/k, where h is the external convective heat transfer coefficient, L the dimension of the problem and k the thermal conductivity of the solid material. Its physical meaning could be seen as the ratio of conductive to convective heat resistance.



Snapshot of an ingot cooling down with two different external convective heat transfers applied on its external faces. In the upper case the value of the Biot number is 13.0 (h value of 10'000 W/m^{2}K) while in the lower case the Biot value is 0.13 (h value of 100W/m^{2}K). It is seen (top pictures) that with a Biot number of 13, there is a thermal gradient in the part and that this gradient is present during the whole cooling. In the opposite case, there is almost no thermal gradient during cooling. Graphs on the right show the evolution of the temperature profile. The top curve does represent the temperature distribution at time t=0, curves below show the evolution of the temperature distribution.

Let's imagine now a simple application in investment casting where the shell is cracking because of stresses due to non-uniform temperature distributions. We want to lower the Biot number to make the temperature distribution more uniform. We cannot reduce L by making the shell thinner. If we try to make it stronger by making it thicker, then the temperature difference across it will only increase and make failure more likely. We may be able to choose a different shell material with a higher thermal conductivity. But the most effective thing to do is to try to reduce the heat transfer coefficient, which is done in reality by wrapping the shell with ceramic fibres reflecting much of the radiant heat back to the shell and creating an air film reducing the convective losses.

The Fourier Number

The Fourier number ($Fo = \alpha t / d^2$) is usually defined as the thermal diffusivity α multiplied by a characteristic time t divided by the square of the length through which conduction occurs. This number is used in unsteady-state heat transfer problems. However in physics this number can also be defined in a different way and can be used in the study of unsteady-state mass diffusion problems. It is then equal to the product of the diffusion coefficient D and a characteristic time divided by the square of a characteristic length. In both cases it can be understood as the ratio of current time to time to reach steady-state. For example, if we place an Al object (typical thermal diffusivity of 7 x 10^{-5} m^2/s) of size of 1.0 m in an oven, we can expect to have a uniform temperature distribution after 7 x 10^5 seconds or 19.4 hours (value of Fourier number of 1).

Let's apply this approach to calculate some characteristic diffusion times in solidification for an aluminium alloy. The thermal diffusivity value is around 10^{-5} m^2/s, the value of solute diffusivity in liquid is around 10^{-9} m^2/s while the value of solute diffusivity in solid is around 10^{-13} m^2/s. For a characteristic length of 1.0 mm, characteristic diffusion times for Fourier numbers equal to one are given in the Table below.

Characteristic diffusion times			
Length	$t(\alpha)$	$t(D_l)$	$t(D_S)$
1mm	0.1 s	16 minutes	19 years

The meaning of these typical values is clear: thermal steady-state is reached in a very short time while steady-state in the liquid can be reached in 16 minutes. However steady-state in the solid can never be reached. This is why it is hopeless to try to smooth differences in composition due to macrosegregation by heat treatment. In order to have a flat profile over a length of 1.0 mm only, a typical treatment time is of 19 years! Solute does not diffuse in the solid state.